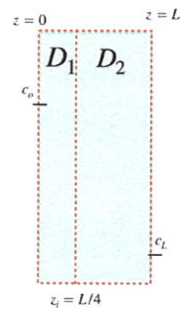
ChE 402: Diffusion and Mass Transfer
Homework 1

1) A chemical company intends to manufacture a 1 cm thick plates of steel for the construction of hyperloop tunnel. To be effective, the plates need to be treated at 1000 °C with H such that the concentration of H is at least 20% of the surface concentration of H at a depth of 200 μm below the surface. H diffuses in steel at 1000 °C with diffusion coefficient of 2.4×10^{-6} cm^2/s. Is it economically feasible to producible such steel. In this production, it becomes too expensive to maintain 1000 °C for more than 1 minute. **(40 points)**

2) Derive and then draw the concentration profile in a composite film (interface at L/4) shown on the right assuming steady-state diffusion. The total thickness of the film is L. Diffusivity of the film on the left is D_1 and on the right is D_2. The concentration on the left side of the film (z=0) is c_0 and on the right side of the film (z=L) is 0. **(30 points)**
The following relationship between D_1 and D_2 is known: $D_1/D_2 = 0.5$

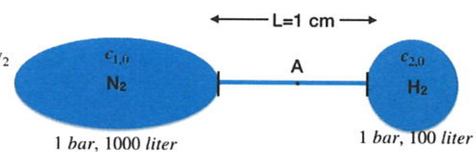


3) In the diffusion apparatus shown here, a large, well-mixed N_2 reservoir is separated from a well-mixed H_2 reservoir using a small tube with length of 1 cm. D for H_2 is 0.2 cm^2/s and for N_2 is 0.1 cm^2/s. Calculate mass, molar and volume average velocities at point A (middle of the tube) at the steady state. Also calculate the flux of N_2 at point A. The cross-section of tube is so small that the concentration in the two reservoirs do not change significantly (a pseudo steady-state can be assumed). **(40 points)**

N_2 is species 1, H_2 is species 2

v = average velocity of moles $\neq 0$

v^m = average velocity of mass $\neq 0 = w_1v_1 + w_2v_2$



(1) $D = 2.4 \times 10^{-6}\ cm^2/s$

To check feasibility, we need to find concentration at

$z = 200\ \mu m$ at time = 1 min

$$\frac{C - C_s}{C_\infty - C_s} = \operatorname{erf} \xi \quad , \quad C_\infty = 0$$

$$\Rightarrow \frac{-C}{C_s} + 1 = \operatorname{erf} \xi \quad \Rightarrow \boxed{\frac{C}{C_s} = 1 - \operatorname{erf} \xi} \text{ — (1)}$$

$$\xi = \frac{z}{\sqrt{4Dt}} = \frac{200 \times 10^{-6}}{\sqrt{4 \times 2.4 \times 10^{-10} \times 60}} = 0.833$$

$$\Rightarrow \operatorname{erf} \xi = 0.76$$

Applying this to (1), we get

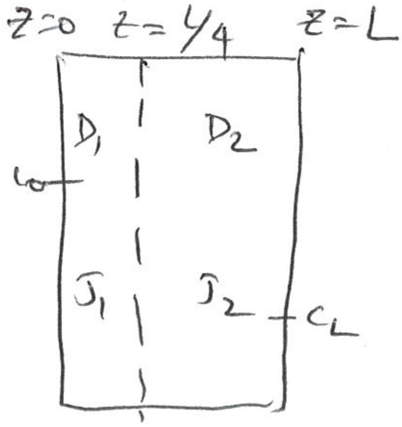
$$\frac{C}{C_s} = 0.24 \quad \Rightarrow \quad C = 0.24\, C_s$$

$\Rightarrow$ Concentration of H at depth of $200\ \mu m$ at 1 min reaches 24 % of the surface concentration

$\Rightarrow$ Process is feasible.

(2) Steady state diffusion

$\Rightarrow J_1 = J_2 = J$



lets say concentration at the interface $= c_i$

$$\Rightarrow D_1\left(\frac{c_0 - c_i}{L/4}\right) = D_2\left(\frac{c_i - c_L}{3L/4}\right)$$

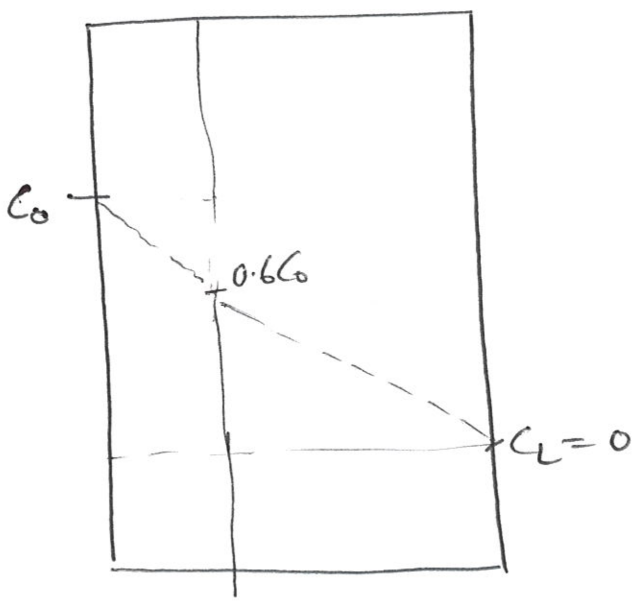
$$\Rightarrow \frac{c_0 - c_i}{c_i - c_L} = \frac{1}{3}\left(\frac{D_2}{D_1}\right)$$

$c_L = 0,\quad D_2/D_1 = 2$

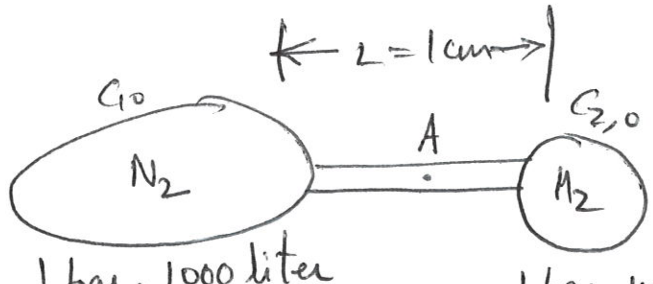
$$\Rightarrow \frac{c_0 - c_i}{c_i} = \frac{2}{3} \Rightarrow 2c_i = 3c_0 - 3c_i$$

$$\Rightarrow 5c_i = 3c_0$$

$$\Rightarrow c_i = 0.6\,c_0$$



(3) Using average velocity of volume



$n_1 = j_1^v + c_1 v^v$

where $v^v = 0$ (no volume change in the system which is closed)

$\Rightarrow \quad n_1 = j_1^v = -D_1 \nabla C_1 = \dfrac{D_1 C_{10}}{L} = \text{constant}$

At point A, $n_1 = C_1 v_1$ where $C_1 = \dfrac{C_{10}}{2}$ based on the symmetry

$\Rightarrow \quad \dfrac{D_1 C_{10}}{L} = C_1 v_1 \quad \Rightarrow v_1 = \dfrac{D_1 C_{10}}{L C_1} = \dfrac{2D_1}{L}$

$\Rightarrow \quad v_1 = \underrightarrow{0.2\ cm/s}$

Similarly $\quad n_2 = \dfrac{D_2 C_{20}}{L} = \text{constant}$

At point A $\quad n_2 = C_2 v_2 \quad$ where $C_2 = \dfrac{C_{20}}{2}$

$\Rightarrow \quad v_2 = \dfrac{D_2 C_{20}}{L C_2} = \dfrac{2D_2}{L}$

$\Rightarrow \quad v_2 = \underleftarrow{-0.4\ cm/s}$

given $y_1 = y_2 = 0.5$ $\underline{\text{at point A}}$ $\Rightarrow v = y_1 v_1 + y_2 v_2$

$= \underleftarrow{-0.1\ cm/s}$

$v^m = w_1 v_1 + w_2 v_2 \quad$ where $w_{1,A} = \dfrac{28}{30}$, $w_{2A} = 2/30$

$\Rightarrow v^m$ at $A = \dfrac{28}{30} \times 0.2 - \dfrac{2}{30} \times 0.4 = \underrightarrow{0.16\ cm/s}$